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Examining Implicit Discretization in Spectral Schemes

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Abstract

Two of the primary reasons rainbow color maps are considered ineffective trace back to the idea that they implicitly discretize encoded data into hue-based bands, yet no research addresses what this discretization looks like or how consistent it is across individuals. This paper presents an exploratory study designed to empirically investigate the implicit discretization of common spectral schemes and explore whether the phenomenon can be modeled by variations in lightness, chroma, and hue. Our results suggest that three commonly used rainbow color maps are implicitly discretized with consistency across individuals. The results also indicate, however, that this implicit discretization varies across different datasets, in a way that suggests the visualization community's understanding of both rainbow color maps, and more generally effective color usage, remains incomplete.

CCS Concepts

• ***Human-centered computing*** → ***Empirical studies in visualization;***

1. Introduction

Two of the primary reasons rainbow color maps are considered harmful stem from an argument that they implicitly discretize encoded data into hue-based bands [BT07,BRT95,Mor09]. The literature argues that this perceived banding both highlights non-existent relationships in the data through the creation of false boundaries and masks real relationships within a given band [BT07]. Our current understanding of the implicit discretization in rainbow color maps, however, is based on a combination of generalized knowledge about how humans perceive the visible spectrum [RLK92] and anecdotal evidence that has yet to be empirically tested [BT07].

To our knowledge, no work has empirically evaluated the perceived banding in rainbow color maps or characterized the potential differences across color scales, datasets, or individuals. Understanding and characterizing any perceived banding in rainbow color maps is important, as precise knowledge of how people perceive these bands is essential for leveraging implicit discretization in color-map design to improve the performance of some tasks [PQMC17,DPR*18]. An improved understanding of implicit discretization also ensures that the visualization guidance regarding rainbow color maps has proper scientific foundations [Kos16].

To better understand the perceived banding in rainbow color maps in color displays, we conducted an exploratory study aimed at assessing both whether rainbow color maps implicitly discretize data and how implicit discretization varies across different individuals, datasets, and spectral schemes. Participants were shown sets of color mapped visualizations and asked to first count the color categories/boundaries that they perceived and then interactively delineate those categories/boundaries. Participants' delineations were then compared against potential boundary locations derived from variation in the perceptual dimensions of color for each spectral scheme. Although previous work has attributed some perceived bands in rainbow color maps to variation in luminance [BRT95,BT07,Mor09], luminance alone cannot explain the banding perceived in all spectral schemes [KRC02]. This study expands the investigation of banding effects to variations across all three perceptual dimensions of color: lightness, chroma, and hue.

The study results suggest that rainbow color maps *are* implicitly discretized with consistency across individuals. Additionally, the results show correspondences between participants' responses and variation in each perceptual dimension of color. The results also indicate that the discretization produced by a given color map varies in unexpected and unpredictable ways across different datasets, revealing practical challenges for common tasks like drawing comparisons across datasets. Further, the findings suggest that the visualization community's current understanding of both rainbow color maps, and more generally effective color usage, remain incomplete.

The remainder of this paper is outlined as follows. Section 2 summarizes related work conducted in the visualization, vision science, and cognitive science communities. Section 3 discusses the wide range of definitions for the term *rainbow color map*. Section 4 details both the study's aims and methods. Section 5 then outlines the results of the study, which we discuss further in Section 6, before summarizing our conclusions in Section 7.

2. Related Work

This paper builds on work from two distinct bodies of literature: the visualization community's prior work regarding rainbow color maps and work regarding the categorical perception of color, conducted predominantly by the vision and cognitive science communities. In this section, we highlight closely related work from both.

2.1. Rainbow Color Maps

Rainbow color maps are claimed to be harmful for three primary reasons. First, because hue is not inherently ordered [War12], ordered relationships in data visualized using rainbow color maps are not necessarily preserved [Mor09, BT07]. The other two reasons are that rainbow color maps both mask and overaccentuate small data differences, biasing our understanding of the underlying data relationships [BT07, KRPC00]. Insufficient luminance variation in certain portions of rainbow color maps can obscure small details, and hue bands can introduce artifacts such as false boundaries that actively mislead users. Current thought further links both problems to the irregular nature of the implicitly perceived discretization.

Several studies provide empirical evidence for parts of these claims. Color matching experiments by Kalvin et al. [KRPC00] and feature discrimination experiments by Ware et al. [WTS*17] support the idea that insufficient luminance variation hides data variation in the green region of the traditional rainbow color map. Rogowitz and Kalvin show that, even though local subsections of the traditional rainbow color map maintain a luminance-based ordering, the overall color map is not inherently ordered [RK01]. Recent work by Liu and Heer indicates that, when asked to compare the relative distances of colors within a color map, respondents are slower and more error prone with the *jet* color map compared to both single-hue and nonrainbow multihue schemes. Further, a study Borkin et al. conducted with medical experts shows that the traditional rainbow color map is ineffective for real-world tasks [BGP*11]. This result is echoed in a recent study Dasgupta et al. ran with climate scientists, where the jet color map produced larger errors in average magnitude comparisons of geospatial maps compared to color maps with monotonic luminance [DPR*18].

Other work suggests rainbow color maps are not always a bad choice. Experiments by both Ware [War88] and Reda et al. [RNA18] show that rainbow color maps are accurate for quantity estimation tasks and provide support for some form-comprehension and gradient-estimation tasks. Additionally, studies by Brewer [Bre97] and Gresh [Gre08] show that modified rainbow color maps are interpreted accurately when used as multihue diverging schemes. Rainbow color map variants that control luminance variation to avoid many of the problems commonly attributed to rainbow color maps also exist [KRC02, Gre11]. Collectively, this work leaves a variety of open questions about if, when, and why rainbow color maps are harmful.

Several papers argue that rainbow color maps are harmful because they implicitly discretize the encoded data into hue-based bands [BT07, BRT95, Mor09]. These bands introduce false boundaries and obscure data variation, thereby leading users *"to infer structure which is not present in the data and to miss details that lie completely within a single color region"* [BRT95]. These arguments, however, are predicated on empirical evidence that visible light is perceptually discretized when diffracted through a prism [RLK92] and on anecdotal examples [BT07]; no work has empirically tested whether rainbow color maps are perceived as banded. Moreover, recent work raises questions about whether and why implicit discretization is problematic. Padilla et al. show that, in grayscale color maps, regularly spaced discretization does not negatively impact, and sometimes improves, accuracy across various tasks [PQMC17], suggesting that discretization can be beneficial. Further, Dasgupta et al. show that although hue banding negatively impacts average magnitude comparisons, it enables more accurate difference comparisons across geospatial datasets [DPR*18].

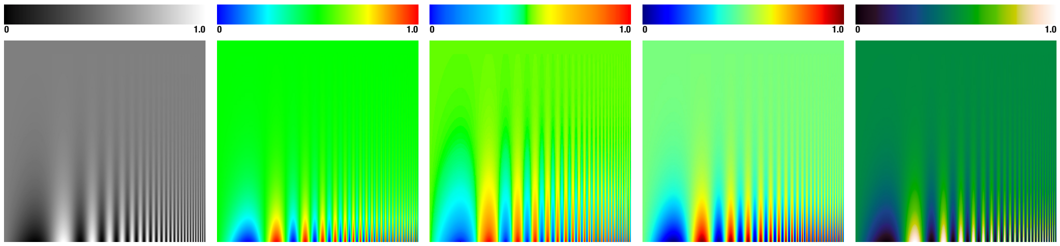
No prior work, however, has empirically investigated the implicit discretization in visualizations using rainbow color maps. We address this gap by exploring the questions of whether rainbow color maps implicitly discretize encoded data into perceived bands and how that discretization varies across individuals, data characteristics, and spectral schemes. Our broad goal is to understand whether any perceived banding in rainbow color maps can be predicted using the perceptual dimensions of color: lightness, chroma, and hue.

2.2. Categorical Perception of Color

The idea that humans perceive continuous color as discretized traces back centuries to experiments by Issac Newton and Hermann von Helmholtz [SCA94]. The phenomenon is currently believed to be an effect of *categorical perception*, where viewers are faster and more accurate at discriminating colors in different categories (e.g., green and blue) compared to colors in the same category (e.g., different shades of blue) [Han16]. Categorical perception represents one possible explanation for why and how people might implicitly discretize rainbow color maps.

An extensive body of literature has investigated categorical perception both in general [GH10] and specifically as it applies to color [BG17, RK09], but little work directly investigates the effects of categorical perception across continuous color ranges. Most existing research focuses on probing individuals' perception at a small set of established color-category boundaries in order to test theories about the perceptual or cognitive underpinnings of color categorical perception, often using pair-wise color judgments [OD02, HHZ*14, WK12]. This focus likely stems from an ongoing debate about the roles of language and perception in forming color categories. One side argues that a perceptual phenomenon gives rise to the formation of consistent categories across languages; the other argues that prior knowledge of language biases individuals' perception. The literature has yet to reach a consensus regarding what drives categorical perception [RK09, Wit18].

Two studies that examine how people perceive color categories across the visible spectrum or approximations thereof present some evidence of consistency in subjects' perception of color category boundaries [SCA94, WG13]. Small sample sizes and specific experimental design choices, however, create questions about the generalizability of these results. Smeulders et al. asked 5 participants to delineate a diffracted spectrum into a specified number of categories [SCA94], and Witzel and Gegenfurtner asked 10 participants to name isolated color samples derived from isoluminant hue circles presented on computer monitors using basic color terms [WG13]. It is not clear that either set of results reflects the potential banding expected in rainbow color mapped visualizations. Moreover, neither study addresses the question of whether categorical perception creates an implicit discretization that might affect



(a) grayscale (b) traditional (c) Gresh (d) jet (e) Kindlmann

Figure 1: *Campbell-Robson contrast sensitivity charts visualized using (a) grayscale, (b) the traditional rainbow color map, (c) Gresh's perceptually linearized rainbow, (d) the jet color map, and (e) the Kindlmann color map show pronounced differences in the extents to which rainbow color maps capture data variation. In each image, spatial frequency increases left to right, and contrast increases bottom to top.*

how a user completes a given task. A handful of visualization papers leverage categorical perception either to create more effective visualizations [Hea96, CSH08, HS12] or to model participant responses [LH18], but no work directly addresses the potential role of categorical perception in discretizing continuous color scales.

3. Defining a Rainbow

Within the visualization community, a variety of color maps that approximate the visible spectrum are described broadly using the term *rainbow color map*. Specific research results, however, often relate to particular spectral schemes [War88, BGP*11, RK01, LH18, DPR*18], raising questions about the generalizability of those results to the larger class of rainbow color maps. This ambiguity is problematic because not all rainbow color maps suffer from the same problems to the same extent.

Figure 1 shows Campbell-Robson contrast sensitivity charts encoded using a variety of rainbow color maps. Contrast sensitivity charts are commonly used to illustrate that rainbow color maps hide data variation [BT07, Mor09, KES13], although prior comparisons are limited to grayscale (Figure 1a) and the *traditional* rainbow color map (Figure 1b). The traditional rainbow color map is commonly defined by tracing the boundary of the device-dependent RGB gamut from blue to red [KRPC00], although a variant that cycles from magenta to red also exists [War88]. Figure 1 also includes comparisons to the *jet* color map from MATLAB [Edd14] (Figure 1d); Gresh's perceptually linearized rainbow [Gre08] (Figure 1c); and the *Kindlmann* color map [KRC02, Mor16] (Figure 1e), which modifies the traditional rainbow color map to linearly increase in perceived luminance.

Figure 1 highlights pronounced differences in the extents to which these color maps capture data variation. Similar differences can also be observed across prior work. Both Kalvin et al. [KRPC00] and Ware et al. [WTS*17] found evidence of low discriminability in the isoluminant, green region of the traditional rainbow color map, whereas Liu and Heer [LH18] found evidence of high discriminability in the corresponding isoluminant, green region of the jet color map. These differences illustrate a need for increased precision in discussions regarding rainbow color map research. Within the context of this paper, the term *rainbow color map* refers to any of the general class of spectral schemes shown in Figure 2. Individual color maps, such as those referenced in our study, are referred to using specific names: the traditional rainbow color map, the jet color map, etc.

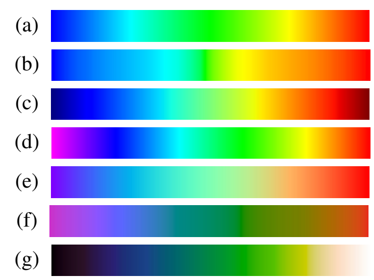


Figure 2: *The term* ***rainbow color map*** *can refer to a variety of spectral schemes that do not necessarily suffer from the same problems to the same extent. Notable examples include: (a) the traditional rainbow color map (truncated at blue), (b) Gresh's perceptually linearized rainbow, (c) the jet color map popularized by* MATLAB*, (d) the traditional rainbow color map (cycling to magenta), (e) the rainbow color map specified by matplotlib, (f) Kindlmann's isoluminant rainbow, and (g) the Kindlmann color map.*

4. Methods and Aims

The primary objective of this research is to understand the nature of implicit discretization in rainbow color maps. If the perceived banding is linked to categorical perception, how much variation should we expect across individuals and how does that variation change across different rainbow color maps? Moreover, given the known effects of spatial frequency on our perception of color-mapped data [KRPC00, RNA18], to what extent should we expect any perceived banding to be affected by the encoded data? To better understand these relationships, we conducted an exploratory study designed to generate empirical observations about how individuals perceive hue bands across different datasets visualized using various rainbow color maps. We focus solely on implicit discretization in electronic displays, while controlling for expected real-world confounds such as gamut differences and viewing conditions. The following subsections detail our hypotheses, stimuli, experimental apparatus, tasks, procedures, and participant demographics.

4.1. Hypotheses

In this study, we explored three main hypotheses.

H1. *In line with the long-standing suppositions of the visualization community [BT07, BRT95, Mor09], we expect that data visualized using rainbow color maps is perceived as implicitly discretized into hue-based bands. Additionally, we predict that this implicit discretization will vary across different rainbow color maps.*

The categorical perception literature provides evidence that humans perceive specific sets of colors as grouped or categorized according to various color terms. When white light is diffracted into a prismatic spectrum, people consistently perceive continuous ranges of wavelengths as individual bands of uniform color [SCA94]. Similar categories also exist in how people perceive and group individual samples of uniform color [HHZ*14, WK12, WG13]. Therefore, we have reason to expect that this same phenomenon might affect visualizations encoded using continuous color maps.

The literature also suggests that the categorical perception of color is inherently tied to a relatively small set of basic color terms [RK09, BK99] that effectively partition color space. By definition, different spectral schemes trace fundamentally different paths through color space, suggesting that they also trace different paths through the regions of color space associated with these color terms. Thus, we expect that the widths of the perceived bands associated with a given basic color term should vary across different rainbow color maps. It is not immediately clear, however, to what extent individual variation might impact this.

H2. *We predict that the implicit discretization produced by a given color map is based on more than just luminance variation.*

The idea that sudden shifts in luminance should cause visible discontinuities in rainbow color maps is well documented in prior work [BRT95, Mor09], but luminance alone cannot explain all the banding seen in common rainbow color maps, which can be surmised from Figures 3 and 4. Figure 3 shows the CIELCh lightness (L*), chroma (C*), and hue (h) profiles for four color maps: a perceptual grayscale, the traditional rainbow color map, jet, and the Kindlmann color map. Figure 4 visualizes three datasets using these same color maps. *Lightness* is a measure of perceived luminance, judged relative to a comparably illuminated white [Fai13]. Being based on pair-wise color judgments, CIELCh is not a perfect metric of perceived color differences in continuous color fields; however, a better alternative does not currently exist [Sza18].

Despite encoding linear data, Figures 4g and 4h both appear to show banding induced by the Kindlmann color map. This banding cannot be explained by the Kindlmann color map's linear lightness profile, seen in Figure 3. Figure 3 also indicates that the traditional rainbow color map's lightness profile has only two sudden changes (i.e., cusps). If luminance perception alone drove this phenomenon, Figures 4a and 4b should contain only three distinct hue bands.

We suspect that variation in chroma and, to a lesser extent, hue also contributes to the banding perceived across the first two columns of Figure 4. Chroma, like saturation, is a relative measure of colorfulness, the distinction being that chroma, similar to lightness, is measured relative to the brightness of a comparably illuminated white, whereas saturation is measured relative to the stimulus' brightness [Fai13]. Figure 3 indicates that the lightness and chroma profiles for both the traditional rainbow and jet color maps have *cusps*, sharp features where two curves intersect, at the exact same locations. The chroma profile for the Kindlmann color map also contains cusps, which could explain perceived banding independent of luminance. Further, *inflection points* or concavity changes in the chroma profiles of each color map appear to loosely correspond to additional hue-band boundaries, such as potential red:orange and blue:light-blue boundaries in both the traditional rainbow and jet color maps. Weaker evidence suggests that hue variation may also be contributing to this phenomenon. Figure 3 indicates that *some* of the cusps in chroma correspond to cusps in hue for both the jet and Kindlmann color maps. Additionally, there are two inflection points in the hue profile for the traditional rainbow color map that might also impact perceived banding. Given these observations, we hypothesize that the implicit discretization in rainbow color maps might be explained by a combination of cusps and inflection points in the perceptual dimensions of color.

H3. *We predict that the implicit discretization perceived in a given visualization will depend on the data being visualized.*

Any banding perceived in a given color-mapped visualization will be related to the color variation in the resulting image. When encoding linear functions, any perceived banding in the resulting image space should be the result of color map artifacts. Thus, assuming rotational invariance, we anticipate that the 1D linear ramp and the 2D radial gradient shown in Figure 4 should have similar perceived bands. With real-world datasets, on the other hand, we expect that the set of the perceived boundaries in image space will reflect a combination of not only color map artifacts but also underlying data features that will vary across datasets. This potential conflation of data features with perceptual artifacts is a core part of why the existing literature argues that rainbow color maps are misleading [BT07]. The larger goal of this hypothesis is to empirically explore the differences between linear functions and real-world datasets in the hope of garnering insights that might allow us to begin to model the perceived banding in rainbow color-mapped visualizations in subsequent work.

4.2. Anticipated Indicators

To explore hypotheses H2 and H3, we derived the locations of cusps and inflection points from the CIELCh lightness, chroma, and hue profiles for each of the rainbow color maps in our study. The resulting locations – which we call *indicators* – are represented as vertical dotted lines in Figure 3. Cusps are modeled as locations of high curvature where curvature magnitude surpasses a specified threshold, and inflection points reflect zero crossings in curvature.

These indicators were derived using standard numerical methods. Using the 256 colors in each color map, we constructed interpolating cubic splines that approximate the CIELCh profiles for each color map. The roots of the second and third derivatives of those splines correspond to the zeros and maxima/minima in curvature, respectively. We threshold the curvature maxima/minima using the absolute value of the second derivative (i.e., curvature magnitude) to generate a set of local maxima/minima with arbitrarily high curvature. Inflection points are derived similarly, using the locations of zero curvature and thresholding based on gradient magnitude. This process is illustrated in Figure 5.

Because numerical differentiation is known to be highly sensitive to small changes [KC01], we employed both Gaussian smoothing on the CIELCh profiles and thresholding of the derivatives to eliminate numeric artifacts generated by noise. We also manually removed any spurious indicators that could be traced to numeric artifacts, such as boundary conditions. An expanded discussion that includes the smoothing and thresholding parameters used to derive the indicators in Figure 3 is included as supplemental material.

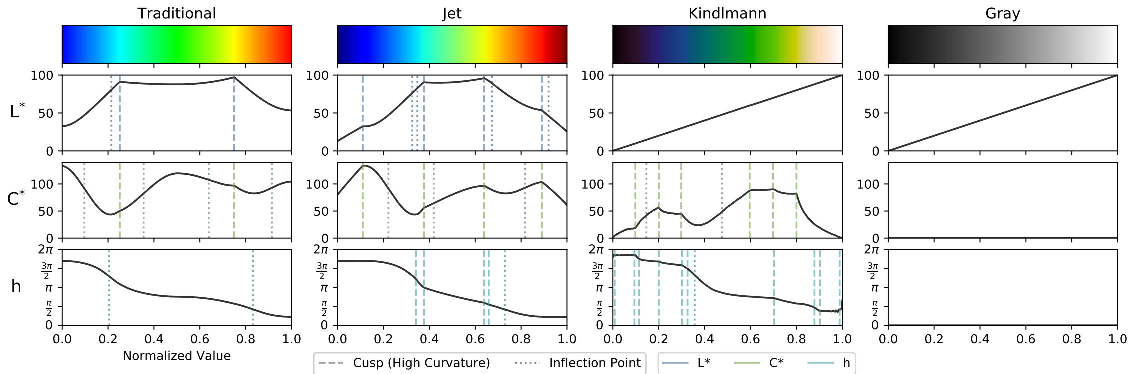


Figure 3: *The CIELCh lightness (L*), chroma (C*), and hue (h) profiles for the four color maps we looked at in our exploratory study, with dotted and dashed lines showing the derived locations of cusps and inflection points.*

4.3. Stimuli

During the study, each participant was presented with 12 stimuli generated by encoding 3 univariate datasets with 4 different color maps, each shown in Figure 4. The datasets included a 1D linear ramp, a 2D radial gradient, and a complex real-world 2D geospatial dataset. The linear ramp and radial gradient datasets are functionally defined as affine transformations of $f(x,y) = x$ and $f(x,y) = x^2 + y^2$, respectively. The complex dataset is a 3-second resolution coastal relief model of Hawaii Island sourced from the National Oceanic and Atmospheric Administration's National Center for Environmental Information [Nat05]. The stimuli shown in Figures 4j to 4l were encoded using a perceptual grayscale color map, created by linearly interpolating from black to white in CIELCh. This color map was chosen as a baseline to enable separating data features from artificial boundaries created by the rainbow color maps, which we anticipate according to hypothesis H3.

The remaining experimental stimuli were generated from 3 rainbow color maps. The traditional rainbow and jet color maps were chosen as well-known and commonly used rainbow color maps. We generated the traditional rainbow color map by linearly interpolating between equally spaced blue, cyan, green, yellow, and red control points in sRGB. For the jet color map, we utilized the implementation included in matplotlib [Hun07].

As a spectral scheme with a linear lightness profile, the Kindlmann color map was chosen to facilitate comparisons between the other rainbow color maps and grayscale. We chose this color map over various other spiral color maps, such as the cubehelix [Gre11] or black body [Mor16] color maps, because it traverses a similar distribution of hue values to the traditional rainbow and jet color maps, it exhibits banding that cannot be explained by luminance variation [DPR*18], and it has an established pattern of use by the visualization community [Mor16, STP17, YLL15, ZH16, DPR*18]. As no accepted device-independent definition of the Kindlmann color map currently exists, we reconstructed the color map directly from the original paper figure, modifying the lightness channel to ensure linearity in CIELCh. An extended discussion of this implementation choice is included as supplemental material.

4.4. Apparatus

The study was conducted in a controlled laboratory setting. All trials were conducted in a windowless room with the lights turned on, using two identically set-up workstations with Dell U2412M monitors. An experimenter ran contrast and gamma monitor tests (http://www.lagom.nl/lcd-test/) prior to the trials, to ensure display constancy. We did not, however, use external color measurement to verify that colors appeared the same on both monitors. At each workstation, the chair, monitor, and keyboard were placed in the same locations for all trials, with a viewing distance of 60 cm and a monitor size of 61 cm (16:10 aspect ratio). Each stimulus was centered full-screen on a medium-gray background, subtending approximately 19.3° in visual angle (768x768 pixels).

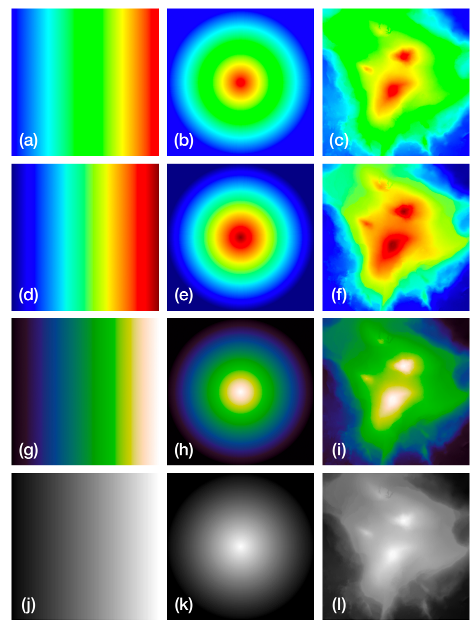


Figure 4: *Experimental stimuli encoding a linear ramp, a radial gradient, and a complex 2D geospatial dataset using four color maps: (a)-(c) the traditional rainbow, (d)-(f)* jet*, (g)-(i) the Kindlmann color map, and (j)-(l) perceptual grayscale.*

4.5. Tasks

For every stimulus, each participant was asked to perform two tasks according to one of two assigned instruction conditions. The first task was to count the number of color categories or color boundaries that they saw. The second was to then interactively delineate those color categories or color boundaries.

Early in the experimental design process, internal discussions revealed that salient features perceived in the yellow and cyan regions of the traditional rainbow and jet color maps could result in two fundamentally different response patterns. As illustrated in Figure 6, an individual could decide to treat these features either as explicit boundaries or as prototypes within larger color categories.

In an effort to ensure that instructions did not disproportionately bias individuals' responses, we developed two separate sets of instructions to capture different ways of completing the tasks. In one set of instructions, participants were asked to make judgments related to *color categories*, which we defined as *"continuous subsets of the color map where colors within the subset are more similar to one another than colors outside the subset."* In the other set of instructions, participants were specifically asked about *color boundaries*, defined as *"the locations where colors on the same side of the boundary are considered more similar to one another than colors on the opposite side of the boundary."* Each participant was assigned a single, consistent instruction set for all trials.

In early experimental prototypes, we also noted that the inclusion of black lines as explicit delimiters appeared to influence judgments about the underlying color category boundaries. This observation is not entirely surprising given that prior work showed that black line delineations increased the number of distinct colors perceived in a diffracted spectrum [SCA94]. It did, however, present a challenge in terms of interface design. Our goal was to understand where people perceive bands, yet the most direct interface for interrogating that question influences the perception of that phenomenon.

We opted to provide an interface where the delimiters covered only part of the underlying experimental stimuli, as illustrated in Figure 6. By requiring participants to count the color categories or boundaries before delineation, we prime each individual's delineation responses. The interface then allows participants to attempt to line up the edge of each delimiter with the boundaries perceived in the undelineated portion of the stimuli. Delimiters can be placed or re-selected by clicking, moved by dragging, or deleted with a double click. Although this design does not entirely control for the potential confounding effects of explicit delimiters, we felt it was satisfactory for an exploratory study.

4.6. Procedure

Each participant provided informed consent before beginning the study. The participant was then assigned one of the two instruction conditions and given a corresponding training module designed to familiarize them with the definitions and interactions in the study.

Upon completion of the training module, the participant was presented with the 12 experimental stimuli using a randomized block scheme. We used 4 blocks, each containing the 3 stimuli encoded using a given color map. Each participant encountered these 4 blocks in a different random order; and within each block, the 3 stimuli were presented in a different random permutation. This procedure resulted in a counterbalanced randomization scheme where each participant encountered exactly 1 of the 24 permutations of the 4 color map blocks, and 4 of the 6 possible dataset permutations across those blocks. For each stimulus, the participants were asked to first count and, subsequently, interactively delineate the color categories or color boundaries that they saw according to their assigned instructions. Working versions of both the study and the training modules are included as supplemental material.

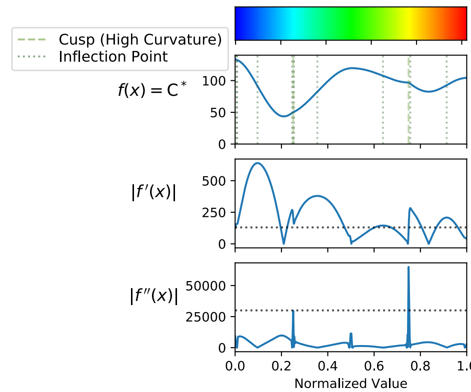
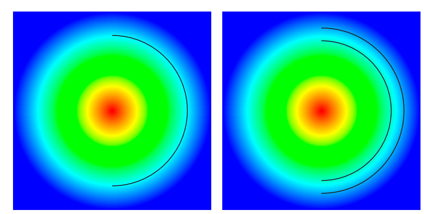


Figure 5: *Deriving the chroma (C*) indicators for the traditional rainbow color map: (upper) the cubic spline approximation of the chroma profile, (center) the derived gradient magnitude, and (bottom) the derived curvature magnitude. Horizontal lines show the thresholds used to isolate the cusps and inflection points, which are represented by the vertical lines overlaid on the chroma profile.*



(a) feature as boundary (b) feature as prototype

Figure 6: *Two fundamentally different response patterns illustrated for the salient cyan feature in the traditional rainbow color map using the study's boundary placement interface: (a) treating the feature as an explicit boundary vs. (b) treating the feature as a prototype subsumed by a larger color category.*

After completing the main study, the participants were asked to fill out a survey in which they answered questions about their judgments during the study and provided demographic information. Although the participants had been prescreened for color vision deficiencies, this survey included explicit secondary checks of their color vision using Ishihara plates along with questions regarding other potential confounds such as prior familiarity with the geography of Hawaii Island. Response times were unconstrained, but the study took most participants about 25 minutes to complete.

4.7. Participants

Participants were recruited from both the University of Utah's psychology participant pool and the University of Utah campus community. They were prescreened for either color vision deficiencies or significant prior exposure to rainbow color maps through the nature of their area of study, and they were compensated for their time at either a rate of \$10/hour or via course credit.

We collected data from 62 participants across both instruction conditions, although we excluded the responses of six individuals who placed more than two standard deviations above the mean number of boundaries from the final analysis. The excluded participants were evenly distributed across our two instruction sets. No additional exclusions were made based on the postexperiment survey responses. Of the 56 participants included in the analysis, 42 were female and 14 were male; and the mean age was 21.55 years (SD = 5.26). The *category* instructions were assigned to 25 participants (23F, 2M), and the *boundary* instructions to 31 participants (19F, 12M). Although there is currently no clear consensus regarding either the presence or absence of sex-related differences in human color vision [JM93, RSHB08, MPMP12] and testing for such differences was outside our intended scope, we recognize that the sex imbalance among our participants is a potential limitation.

5. Results

Given our hypotheses that implicit discretization is occurring and is influenced by both color map and dataset characteristics, we were primarily interested in analyzing where individuals perceive and delineate banding in rainbow color-mapped visualizations. Although our open-ended boundary placement task directly examines this question, we have no way of knowing a priori which subset of participants' delimiters is supposed to correspond to a particular perceived boundary. That correspondence would require prior knowledge of the very facts we are attempting to establish: that individuals perceive bands and where they perceive the boundaries of those bands. It is not clear how one would perform quantitative analyses on predicted cusps and inflection points without prior knowledge of these facts. As a result, much of our analysis relies on qualitative visual analysis methods, which provide a structured way of exploring both participants' response trends and our hypotheses about what drives those trends, free from any assumptions about the existence or nature of hue banding. As a descriptive analysis, we also tested if the color maps influenced the number of color boundaries or categories that participants counted and placed. The statistical analyses are discussed in Section 5.1, and the remaining subsections provide an overview of our visual analyses as they pertain to each of our three hypotheses. We have also included a variety of interactive tools and expanded discussions as supplemental material to assist readers in better assessing the validity of our claims.

5.1. Descriptive Statistical Analysis

To get an initial understanding of the relationships present in the results, we conducted statistical analyses on the number of boundaries/categories participants perceived. We used a linear mixed-effects analysis due to the mixed design with unbalanced sample sizes. Participants were modeled as a random effect, and color map (traditional, grayscale, jet, Kindlmann), dataset (1D, 2D, complex), instruction condition (category, boundary), task/response-method (counted, delineated), and potential color-map:dataset and instruction:response-method interactions were all modeled as fixed effects. Additionally, the grayscale color map and complex dataset were used as reference groups, given that we specifically hypothesized differences compared to these groups.

While full equations and output can be found in the supplemental materials, Figure 7 illustrates the core relationships in this linear mixed-effects analysis. The analysis revealed main effects where each of the rainbow color maps elicited significantly more delineations than grayscale, and the 1D dataset elicited significantly

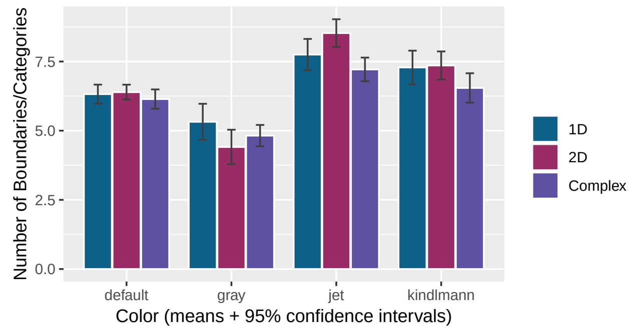


Figure 7: *The mean number of delineations that participants perceived and/or placed along with the 95% confidence interval for each color map and dataset. Descriptive statistical analysis indicates that each rainbow color map elicited significantly more delineations than grayscale, that the 1D dataset elicited significantly more delineations than the complex dataset, and that there were significant color-map:dataset interactions.*

more delineations than the complex dataset. It also showed significant color-map:dataset interactions, with subsequent post hoc analysis revealing that, for both the jet and Kindlmann color maps, participants perceived significantly more boundaries for the 2D dataset compared to the complex dataset but no significant difference in the number of boundaries for grayscale. Additionally, both before and after accounting for these color-map:dataset interactions, neither the instruction condition nor the task/response-method had a significant effect on the number of perceived boundaries.

These statistically significant effects do support the idea that both color map and dataset influence how people perceive boundaries in a given color-mapped visualization, but proving or disproving our hypotheses hinges on showing differences in the distributions of the perceived boundaries. Consider the grayscale results shown in Figure 7, for example. As "black", "white", and "gray" are all basic color terms in English [BK99], meaning grayscale contains multiple color categories, we expect participants to count and delineate boundaries in the grayscale stimuli. We further expect, however, that any delineated boundaries will be randomly distributed in the 1D and 2D stimuli, but centered around data features in the complex stimuli. Likewise, for each rainbow color map, we anticipate that the delineated boundaries will center around color map artifacts in the 1D and 2D stimuli, but be confounded by data features in the complex stimuli. In each case, understanding the distribution of participants' delineations is critical.

5.2. H1: Evidence of Implicit Discretization

Figure 8 provides an overview of the distribution of participants' placed delimiters. For each color map, dataset, and instruction condition, we use kernel density estimation (KDE) to calculate a probability density function (pdf) from the participants' collective delimiter placements. The pdfs shown use different bandwidths, each computed from the associated delimiter placements through multiple iterations of *leave-subject-out* Monte Carlo cross-validation (CV), utilizing a train-test split of 90% to 10%. Leave-subject-out CV is an established blocked CV approach with theoretic optimality that accounts for dependencies within subject responses [XH12, SLJ*17, RBC*17, LVS*17]. Peaks in the resulting

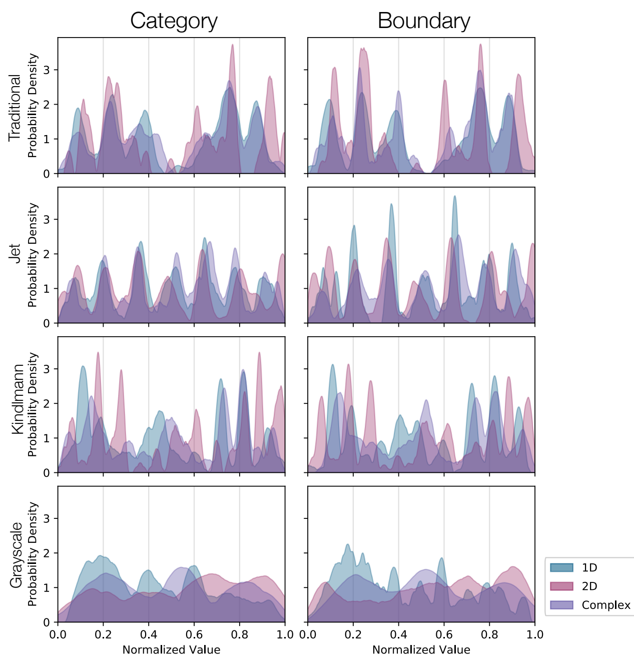


Figure 8: *Probability density functions fit to the participants' collective delimiter placements, partitioned by wording condition, color map, and dataset. Peaks highlight clusters in participants' responses for all three rainbow color maps across all three datasets. Participants' greyscale responses also show a few larger clusters but, overall, are more uniformly distributed.*

pdfs highlight consistencies across participants' placed delimiters. Figure 8 also illustrates that the distributions of participants' delimiters are largely similar across both instruction sets.

For each of the three rainbow color maps, participants' delimiter placements are clustered around distinct locations, but those locations vary both across the color maps for a given dataset and across the datasets for a given color map. In each case, however, the clusters are irregularly spaced, confirming that the perceived bands in rainbow color maps are not uniform in size. By comparison, participants' delimiter placements for the grayscale stimuli are more uniformly distributed across the normalized data value range. Some of the patterns in the grayscale responses, however, can be explained either by artifacts caused by mapping the perceptual grayscale color map into 24-bit RGB color or by data features in the case of the complex dataset. The former is illustrated in the 1D grayscale results presented in Figure 9, where breaking out participants' delimiter placements by individual shows responses clustered around a series of doubled values in the color map. Further discussion of grayscale patterns is included as supplemental material.

Taken together, these results provide empirical support for H1. Participants appear to implicitly discretize rainbow color-mapped datasets with marked consistency across individuals. Moreover, as hypothesized, this discretization varies across the different rainbow color maps tested. Given the provided task, it is possible individuals may have also attempted to use color categories when reasoning about grayscale; however, the clustering in participants' responses is less consistent than for the rainbow color maps.

5.3. H2: Clear Correspondences Beyond Luminance

In comparing participants' responses to the derived indicator sets, the results support hypothesis H2. As we hypothesized, luminance does play a role in the implicit discretization observed in the rainbow color-mapped stimuli, but so does chroma and, to a lesser extent, hue. As shown in Figure 9, the majority of participants' response trends correspond to cusps or inflection points in the CIELCh profiles of each color map. Not every indicator predicts a response trend, however. Here, we provide an overview of participants' responses related to the 1D dataset, where any clusters or trends should be artifacts of the color maps themselves. Supplemental materials show similar findings across each of the datasets and instruction conditions in the study.

For each perceptual dimension of color (lightness, chroma, and hue), cusps in the CIELCh profiles of a given color map exhibit some correspondence with participants' response trends. As prior work predicts [BRT95, Mor09], the cusps associated with the salient cyan and yellow features in both the traditional rainbow (Figures 9a and 9b) and jet color maps (Figures 9d and 9e) correspond to strong participant response trends in 1D. In addition, strong response trends align with some cusps in the chroma and hue profiles of the Kindlmann color map. In other cases, however, cusps in the perceptual dimensions of the color maps have weak or no correspondence with participants' delimiter placements. The coincident lightness and chroma indicators corresponding to the dark-blue:blue boundary in the jet color map (Figure 9c), for example, capture only the right-hand side of a split response trend. Additionally, the 1D Kindlmann results reveal chroma and hue cusps that either correspond to weak trends that emerge only when participants place a large number of boundaries (Figures 9f and 9g) or fail to correspond to any response trends (Figure 9h).

The correspondences between participants' response trends and inflection points in each color map's CIELCh profiles are similarly mixed. Inflection points in chroma capture a number of strong response trends that are not predicted by cusps, such as those corresponding to potential blue:light-blue boundaries in the traditional rainbow (Figure 9i) and jet color maps (Figure 9m). Again, however, not every inflection point corresponds to a response trend. Certain inflection-point indicators exhibit pronounced offsets from their associated response trends (Figures 9j to 9l), whereas others have no corresponding response trend (Figure 9n).

5.4. H3: Unexpected Patterns in Data-Driven Variation

As shown in Figures 10 and 11, for each of the rainbow color maps tested, the different datasets show shifts in the locations and consistency of clusters in participants' responses. Consequently, our results provide support for our H3 hypothesis that implicit discretization depends on the dataset. The variation that we found, however, differs from what we originally anticipated. We expected that a complex stimuli based on real-world data would result in an implicit discretization different from a smoothly varying 1D or 2D stimuli. This reasoning, however, neither predicts nor explains the observed differences in our participants' response trends for the 1D vs. 2D stimuli for each rainbow color map. Further, for the complex stimuli, we found no clear indication that the underlying data features impacted participants' response trends. Neither Figure 10

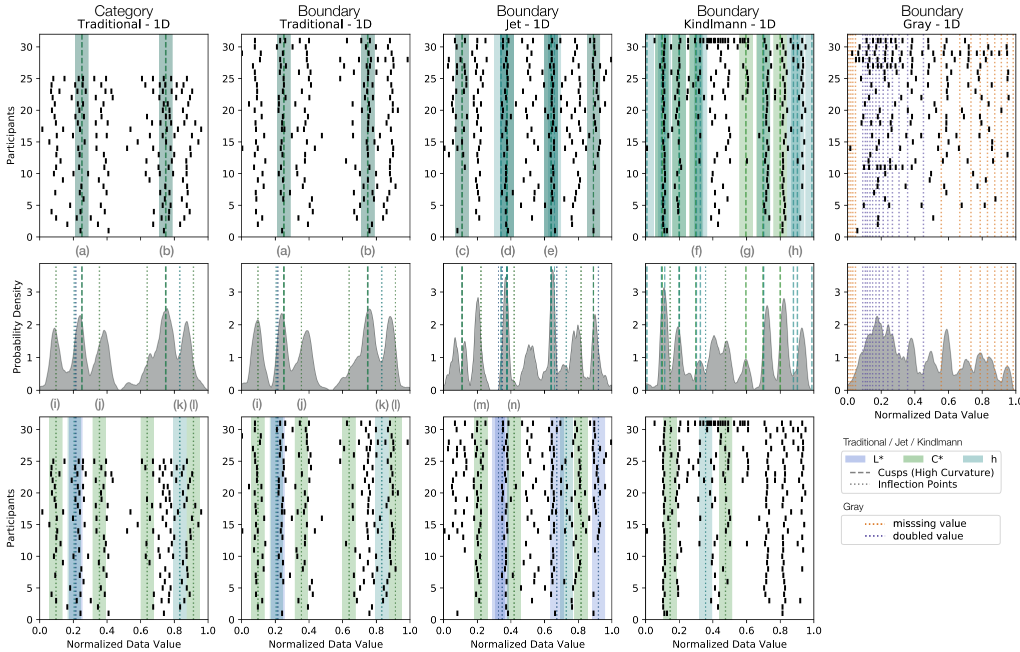


Figure 9: *An overview of participants' delimiter placements in the 1D experimental stimuli. In the top and bottom plots, each row of marks contains the delimiters placed by a single participant with participants ordered along the y-axis by the average number of delimiters they placed overall. For each rainbow color map stimuli, dotted lines show the locations of cusps (top) and inflection points (bottom), with corresponding bands showing the expected individual variation for color category boundaries [WK12]. The same indicators are also overlaid on the pdfs (center) estimated from the delimiters. Convenience labels (a)-(n) are included for indicators referred to in the text. For the grayscale stimuli, dotted lines mark the locations of color map artifacts, with doubled values corresponding to a large response cluster.*

nor Figure 11 provides evidence of data features creating or accentuating perceived boundaries.

Figure 10 provides an overview of participants' delimiter placements across the 3 experimental stimuli encoded using the traditional rainbow color map. This overview contains several notable differences in the strengths and/or locations of participants' response trends. The response trend associated with the leftmost inflection-point indicator (Figure 10a) shifts to the right in the 2D stimuli compared to 1D stimuli, but dissipates into a weaker trend in the complex stimuli. The other end of the color map (Figure 10d) exhibits a pronounced shift in the location of participants' 1D and 2D response trends. Also, toward the center of the color map 2 more trends (near Figures 10b and 10c, respectively) vary in strength across the 3 datasets. Even though each of these trends happens to correlate with an inflection point in chroma, these shifts do not appear specific to trends associated with either chroma variation or inflection point indicators. Figure 11 exhibits similar variations across both the jet and Kindlmann stimuli that affect trends corresponding to a wide variety of indicator types.

In summary, the results do support H3, but they also highlight questions about the nature of the interaction between color maps and datasets. Additional research is needed to determine which dataset characteristics produce variation in implicit discretization and what the underlying perceptual mechanisms for this effect are.

6. Discussion

The perceived banding in rainbow color-mapped visualizations depends on the data being encoded, but in a way that is neither predicted nor readily explained by existing theory. The results presented in this paper show differences in participants' discretizations of smoothly varying linear and radial data gradients that cannot be immediately explained by the human visual system's decreased chromatic sensitivity to high-spatial-frequency information [KRPC00,WTS*17,RNA18]. Given that both the 2D and complex datasets contain varying gradient magnitudes, known interactions between color and size might account for the data-driven variation we observed. Models for color-size effects [Sza18], however, have not yet been extended to handle the complexities of continuous scalar fields.

The results also provide evidence that implicit discretization is driven by more than just luminance. We illustrate correspondences between the perceived banding in rainbow color maps and both cusps and inflection points in each of the perceptual dimensions of those color maps. The rainbow color maps that we tested, however, contain coincident and proximately located indicators, making it challenging to fully separate the effects of luminance, chroma, and hue. Assessing what truly drives many of the individual response trends that we observed would require more systematic control than was present in our exploratory study.

The results further indicate that the implicit discretization caused by rainbow color maps is relatively consistent across individuals. Although the nature of the study's tasks did not allow us to directly assess the amount of individual variation across participants' perceived hue-bands, estimates of individual variation from prior color category experiments [WK12] approximate the variation in many

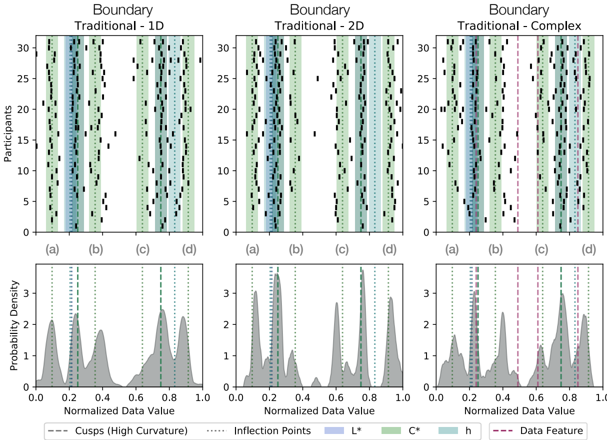


Figure 10: *An overview of participants' boundary placements within the traditional rainbow color map stimuli, showing changes in participants' response trends across the different datasets. Contrary to expectations, the changes include noticeable shifts in responses between 1D and 2D. Further, the underlying data features in the complex dataset appear to have had minimal impact on participants' responses. Convenience labels (a)-(d) are included for specific indicators referred to in the text.*

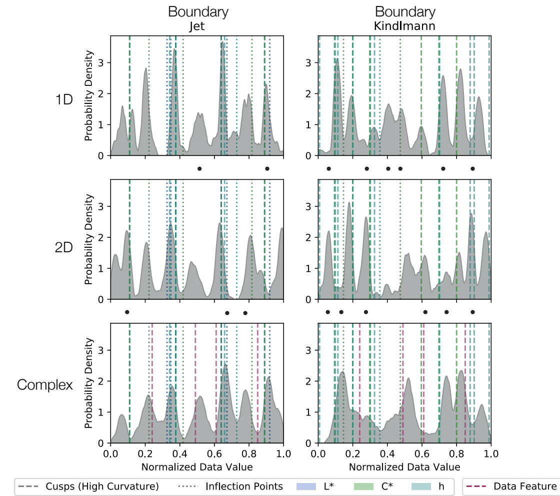


Figure 11: *The probability density plots of participants' delimiter placements for both the jet and Kindlmann stimuli also exhibit significant variation in participants' response trends across all three datasets. Notable differences between the adjacent plots are marked with a ● symbol.*

of the response trends seen in the results. The results also confirm that the perceived hue-bands in rainbow color maps are, indeed, irregularly spaced. We observed no apparent confounding impacts from sex differences or inter-monitor variation, though both are potential limitations that should be addressed in subsequent work.

The study's findings show that different datasets create unpredictable variation in the perceived hue bands in rainbow color-mapped visualizations. This unpredictability presents challenges for experts in a variety of scientific fields [BGP*11, QM16, BTGM16, KES13, ZDM*15, DPR*18], where the implicit discretization in rainbow color maps is used either for classification or as a heuristic for quick visual comparisons. Moreover, given that the results show similar data-driven inconsistencies in the Kindlmann color map, which follows the visualization community's core guidelines regarding effective color usage [BRT95], these same practical challenges may apply to a larger set of multi-hue continuous color scales. This variation could also explain the recent finding of Dasgupta et al. that hue banding negatively impacted magnitude estimation [DPR*18].

Despite the visualization community's promotion of more perceptually appropriate alternatives [BGP*11, BRT95, Mor09, Tru81, LH92, Gre08, KRC02], rainbow color maps remain commonplace in a variety of scientific domains, including medicine [BGP*11], atmospheric and climate sciences [QM16, DPW*15], bioengineering [BTGM16], aerospace [KES13], and astronomy [ZDM*15]. Although domain convention is often used to justify the inclusion of rainbow color map variants in visualization systems [WP13, QM16, PWB*09], we still do not understand *why* experts continue to gravitate to spectral schemes. Cited reasons include familiarity [BGP*11, QM16], aesthetic preference [BGP*11, Bre97, Mor16], and ease of use [BT07, Mor16], but evidence also suggest that rainbow color maps may be a *satisficing* design choice for specific types of tasks, such as locating and quantifying extreme values [DPW*15, WTS*17, WTB*18, RNA18, War88, WTB*18]. Improving our understanding of both how rainbow color maps are used and the ways in which they are ineffective could lead to improved guidance regarding effective color usage more broadly.

7. Conclusions and Future Work

In this paper, we presented an exploratory study investigating the nature of hue banding in rainbow color maps. The results represent a necessary first step in addressing open questions, including whether rainbow color maps implicitly discretize encoded data into hue-based bands and how that discretization varies across different individuals, datasets, and spectral schemes. The results presented in this paper also suggest that the visualization community's current understanding of how rainbow color maps are perceived and used remains incomplete.

The results begin to address gaps in our understanding of the nature of implicit discretization in common spectral schemes, but they also reveal open questions. Rainbow color maps appear to discretize data into hue-based bands, but we currently have an insufficient understanding of the mechanisms that drive this phenomenon and no method for modeling or predicting the banding. Additional potential directions for future work include exploring whether the gradient variation in the encoded datasets plays a role in the response trend variations we observed, examining how to minimize implicit discretization in multihue color maps, and attempting to compare the effects of implicit discretization to the effects of explicit discretization as tested by Padilla et al. [PQMC17].

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References

[BG17] BROGAARD B., GATZIA D. E.: Cortical Color and the Cognitive Sciences. *Topics in Cognitive Science 9*, 1 (Jan. 2017), 135–150. doi:10.1111/tops.12241. 2

[BGP*11] BORKIN M., GAJOS K., PETERS A., MITSOURAS D., MELCHIONNA S., RYBICKI F., FELDMAN C., PFISTER H.: Evaluation of Artery Visualizations for Heart Disease Diagnosis. *IEEE Transactions on Visualization and Computer Graphics 17*, 12 (Dec. 2011), 2479–2488. doi:10.1109/TVCG.2011.192. 2, 3, 10

[BK99] BERLIN B., KAY P.: *Basic Color Terms: Their Universality and Evolution*. Center for the Study of Language and Inf, Stanford, Calif, Mar. 1999. 4, 7

[Bre97] BREWER C. A.: Spectral Schemes: Controversial Color Use on Maps. *Cartography and Geographic Information Systems 24*, 4 (Jan. 1997), 203–220. doi:10.1559/152304097782439231. 2, 10

[BRT95] BERGMAN L. D., ROGOWITZ B. E., TREINISH L. A.: A Rule-based Tool for Assisting Colormap Selection. In *IEEE Conference on Visualization, 1995. Visualization '95.* (Washington, DC, USA, Oct. 1995), IEEE Computer Society, pp. 118–125, 444. doi:10.1109/VISUAL.1995.480803. 1, 2, 3, 4, 8, 10

[BT07] BORLAND D., TAYLOR R. M.: Rainbow Color Map (Still) Considered Harmful. *IEEE Computer Graphics and Applications 27*, 2 (Mar. 2007), 14–17. doi:10.1109/MCG.2007.323435. 1, 2, 3, 4, 10

[BTGM16] BURTON B. M., TATE J. D., GOOD W., MACLEOD R. S.: The role of reduced left ventricular, systolic blood volumes in ST segment potentials overlying diseased tissue of the ischemic heart. In *2016 Computing in Cardiology Conference (CinC)* (Sept. 2016), pp. 1–4. doi:10.23919/CIC.2016.7868716. 10

[CSH08] CHUANG J., STONE M., HANRAHAN P.: A Probabilistic Model of the Categorical Association Between Colors. *Color and Imaging Conference 2008*, 1 (Jan. 2008), 6–11. 3

[DPR*18] DASGUPTA A., POCO J., ROGOWITZ B., HAN K., BERTINI E., SILVA C. T.: The Effect of Color Scales on Climate Scientists' Objective and Subjective Performance in Spatial Data Analysis Tasks. *IEEE Transactions on Visualization and Computer Graphics* (2018), 1–1. doi:10.1109/TVCG.2018.2876539. 1, 2, 3, 5, 10

[DPW*15] DASGUPTA A., POCO J., WEI Y., COOK R., BERTINI E., SILVA C. T.: Bridging Theory with Practice: An Exploratory Study of Visualization Use and Design for Climate Model Comparison. *IEEE Transactions on Visualization and Computer Graphics 21*, 9 (Sept. 2015), 996–1014. doi:10.1109/TVCG.2015.2413774. 10

[Edd14] EDDINS S.: *Rainbow Color Map Critiques: An Overview and Annotated Bibliography*. Tech. Rep. 92238v00, MathWorks, Sept. 2014. 3

[Fai13] FAIRCHILD M. D.: *Color Appearance Models*, 3 ed. Wiley-IS&T Series in Imaging Science and Technology. John Wiley & Sons, Ltd, 2013. 4

[GH10] GOLDSTONE R. L., HENDRICKSON A. T.: Categorical perception. *Wiley Interdisciplinary Reviews: Cognitive Science 1*, 1 (Jan. 2010), 69–78. doi:10.1002/wcs.26. 2

[Gre08] GRESH D.: *Self-Corrected Perceptual Colormaps*. Tech. Rep. RC24542 (W0804-104), IBM, 2008. 2, 3, 10

[Gre11] GREEN D. A.: A colour scheme for the display of astronomical intensity images. *Bulletin of the Astronomical Society of India 39* (June 2011), 289–295. arXiv:1108.5083. 2, 5

[Han16] HANLEY J. R.: Color Categorical Perception. In *Encyclopedia of Color Science and Technology*, Luo M. R., (Ed.). Springer New York, New York, NY, 2016, pp. 239–243. doi:10.1007/978-1-4419-8071-7_61. 2

[Hea96] HEALEY C. G.: Choosing effective colours for data visualization. In *Visualization '96. Proceedings.* (Oct. 1996), pp. 263–270. doi:10.1109/VISUAL.1996.568118. 3

[HHZ*14] HU Z., HANLEY J. R., ZHANG R., LIU Q., ROBERSON D.: A conflict-based model of color categorical perception: Evidence from a priming study. *Psychonomic Bulletin & Review 21*, 5 (Oct. 2014), 1214–1223. doi:10.3758/s13423-014-0603-8. 2, 4

[HS12] HEER J., STONE M.: Color Naming Models for Color Selection, Image Editing and Palette Design. In *Proceedings of the SIGCHI Conference on Human Factors in Computing Systems* (New York, NY, USA, 2012), CHI '12, ACM, pp. 1007–1016. doi:10.1145/2207676.2208547. 3

[Hun07] HUNTER J. D.: Matplotlib: A 2D graphics environment. *Computing In Science & Engineering 9*, 3 (2007), 90–95. doi:10.1109/MCSE.2007.55. 5

[JM93] JORDAN G., MOLLON J. D.: A study of women heterozygous for colour deficiencies. *Vision Research 33*, 11 (July 1993), 1495–1508. doi:10.1016/0042-6989(93)90143-K. 7

[KC01] KINCAID D. R., CHENEY E. W.: *Numerical Analysis: Mathematics of Scientific Computing*, 3 edition ed. Brooks Cole, Pacific Grove, CA, Oct. 2001. 4

[KES13] KIRBY M., ETIENE T., SILVA C.: "Flow Visualization" Juxtaposed With "Visualization of Flow": Synergistic Opportunities Between Two Communities". In *51st AIAA Aerospace Sciences Meeting Including the New Horizons Forum and Aerospace Exposition*. American Institute of Aeronautics and Astronautics, 2013. doi:10.2514/6.2013-1163. 3, 10

[Kos16] KOSARA R.: An Empire Built On Sand: Reexamining What We Think We Know About Visualization. In *Proceedings of the Sixth Workshop on Beyond Time and Errors on Novel Evaluation Methods for Visualization* (New York, NY, USA, 2016), BELIV '16, ACM, pp. 162–168. doi:10.1145/2993901.2993909. 1

[KRC02] KINDLMANN G., REINHARD E., CREEM S.: Face-based Luminance Matching for Perceptual Colormap Generation. In *Visualization, 2002. VIS '02.* (Nov. 2002), pp. 299–306. doi:10.1109/VISUAL.2002.1183788. 1, 2, 3, 10

[KRPC00] KALVIN A. D., ROGOWITZ B. E., PELAH A., COHEN A.: Building Perceptual Color Maps for Visualizing Interval Data. In *Proceedings of SPIE* (2000), vol. 3959, pp. 323–335. doi:10.1117/12.387169. 2, 3, 9

[LH92] LEVKOWITZ H., HERMAN G. T.: Color Scales for Image Data. *IEEE Computer Graphics and Applications 12*, 1 (Jan. 1992), 72–80. doi:10.1109/38.135886. 10

[LH18] LIU Y., HEER J.: Somewhere Over the Rainbow: An Empirical Assessment of Quantitative Colormaps. In *ACM Human Factors in Computing Systems (CHI)* (2018). 3

[LVS*17] LITTLE M. A., VAROQUAUX G., SAEB S., LONINI L., JAYARAMAN A., MOHR D. C., KORDING K. P.: Using and understanding cross-validation strategies. Perspectives on Saeb et al. doi:10.1093/gigascience/gix020. 7

[Mor09] MORELAND K.: Diverging Color Maps for Scientific Visualization. In *Advances in Visual Computing* (Berlin, Heidelberg, Nov. 2009), Springer-Verlag, pp. 92–103. doi:10.1007/978-3-642-10520-3_9. 1, 2, 3, 4, 8, 10

[Mor16] MORELAND K.: Why We Use Bad Color Maps and What You Can Do About It. *Electronic Imaging 2016*, 16 (Feb. 2016), 1–6. doi:10.2352/ISSN.2470-1173.2016.16.HVEI-133. 3, 5, 10

[MPMP12] MURRAY I. J., PARRY N. R. A., MCKEEFRY D. J., PANORGIAS A.: Sex-related differences in peripheral human color vision: A color matching study. *Journal of Vision 12*, 1 (Jan. 2012), 18–18. doi:10.1167/12.1.18. 7

[Nat05] NATIONAL GEOPHYSICAL DATA CENTER: Coastal relief model - Hawaii, 2005. URL: http://dx.doi.org/10.7289/V5RF5RZZ, doi:10.7289/V5RF5RZZ. 5

[OD02] ÖZGEN E., DAVIES I. R. L.: Acquisition of categorical color perception: A perceptual learning approach to the linguistic relativity hypothesis. *Journal of Experimental Psychology: General 131*, 4 (Dec. 2002), 477–493. doi:10.1037/0096-3445.131.4.477. 2

[PQMC17] PADILLA L., QUINAN P. S., MEYER M., CREEM-REGEHR S. H.: Evaluating the Impact of Binning 2D Scalar Fields. *IEEE Transactions on Visualization and Computer Graphics 23*, 1 (Jan. 2017), 431–440. doi:10.1109/TVCG.2016.2599106. 1, 2, 10

[PWB*09] POTTER K., WILSON A., BREMER P.-T., WILLIAMS D., DOUTRIAUX C., PASCUCCI V., JOHNSON C. R.: Ensemble-Vis: A Framework for the Statistical Visualization of Ensemble Data. In *2009 IEEE International Conference on Data Mining Workshops* (Dec. 2009), pp. 233–240. doi:10.1109/ICDMW.2009.55. 10

[QM16] QUINAN P. S., MEYER M.: Visually Comparing Weather Features in Forecasts. *IEEE Transactions on Visualization and Computer Graphics 22*, 1 (Jan. 2016), 389–398. doi:10.1109/TVCG.2015.2467754. 10

[RBC*17] ROBERTS D. R., BAHN V., CIUTI S., BOYCE M. S., ELITH J., GUILLERA-ARROITA G., HAUENSTEIN S., LAHOZ-MONFORT J. J., SCHRÖDER B., THUILLER W., WARTON D. I., WINTLE B. A., HARTIG F., DORMANN C. F.: Cross-validation strategies for data with temporal, spatial, hierarchical, or phylogenetic structure. 913–929. doi:10.1111/ecog.02881. 7

[RK01] ROGOWITZ B. E., KALVIN A. D.: The "Which Blair Project": A Quick Visual Method for Evaluating Perceptual Color Maps. In *Visualization, 2001. VIS '01.* (Oct. 2001), pp. 183–556. doi:10.1109/VISUAL.2001.964510. 2, 3

[RK09] REGIER T., KAY P.: Language, thought, and color: Whorf was half right. *Trends in Cognitive Sciences 13*, 10 (Oct. 2009), 439–446. doi:10.1016/j.tics.2009.07.001. 2, 4

[RLK92] ROGOWITZ B. E., LING D. T., KELLOGG W. A.: Task Dependence, Veridicality, and Preattentive Vision: Taking Advantage of Perceptually Rich Computer Environments. vol. 1666, pp. 504–513. doi:10.1117/12.135996. 1, 2

[RNA18] REDA K., NALAWADE P., ANSAH-KOI K.: Graphical Perception of Continuous Quantitative Maps: The Effects of Spatial Frequency and Colormap Design. In *Proceedings of the 2018 CHI Conference on Human Factors in Computing Systems (to Appear)* (Montreal, QC, Canada, Apr. 2018), CHI '18, ACM. doi:10.1145/3173574.3173846. 2, 3, 9, 10

[RSHB08] RODRÍGUEZ-CARMONA M., SHARPE L. T., HARLOW J. A., BARBUR J. L.: Sex-related differences in chromatic sensitivity. *Visual Neuroscience 25*, 3 (May 2008), 433–440. doi:10.1017/S095252380808019X. 7

[SCA94] SMEULDERS N., CAMPBELL F. W., ANDREWS P. R.: The role of delineation and spatial frequency in the perception of the colours of the spectrum. *Vision Research 34*, 7 (Apr. 1994), 927–936. doi:10.1016/0042-6989(94)90041-8. 2, 4, 6

[SLJ*17] SAEB S., LONINI L., JAYARAMAN A., MOHR D. C., KORDING K. P.: The need to approximate the use-case in clinical machine learning. doi:10.1093/gigascience/gix019. 7

[STP17] SUMMA B., TIERNY J., PASCUCCI V.: Visualizing the Uncertainty of Graph-based 2D Segmentation with Min-path Stability. *Computer Graphics Forum 36*, 3 (2017), 133–143. doi:10.1111/cgf.13174. 5

[Sza18] SZAFIR D. A.: Modeling Color Difference for Visualization Design. *IEEE Transactions on Visualization and Computer Graphics 24*, 1 (Jan. 2018), 392–401. doi:10.1109/TVCG.2017.2744359. 4, 9

[Tru81] TRUMBO B. E.: A Theory for Coloring Bivariate Statistical Maps. *The American Statistician 35*, 4 (1981), 220–226. doi:10.2307/2683294. 10

[War88] WARE C.: Color Sequences for Univariate Maps: Theory, Experiments and Principles. *IEEE Computer Graphics and Applications 8*, 5 (Sept. 1988), 41–49. doi:10.1109/38.7760. 2, 3, 10

[War12] WARE C.: *Information Visualization: Perception for Design*, 3 ed. Morgan Kaufmann Publishers Inc., San Francisco, CA, USA, 2012. 2

[WG13] WITZEL C., GEGENFURTNER K. R.: Categorical sensitivity to color differences. *Journal of Vision 13*, 7 (June 2013), 1–1. doi:10.1167/13.7.1. 2, 4

[Wit18] WITZEL C.: Misconceptions About Colour Categories. *Review of Philosophy and Psychology* (July 2018). doi:10.1007/s13164-018-0404-5. 2

[WK12] WEBSTER M. A., KAY P.: Color categories and color appearance. 375–392. doi:10.1016/j.cognition.2011.11.008. 2, 4, 9

[WP13] WARE C., PLUMLEE M. D.: Designing a Better Weather Display. *Information Visualization 12*, 3-4 (July 2013), 221–239. doi:10.1177/1473871612465214. 10

[WTB*18] WARE C., TURTON T. L., BUJACK R., SAMSEL F., SHRIVASTAVA P., ROGERS D. H.: Measuring and Modeling the Feature Detection Threshold Functions of Colormaps. *IEEE Transactions on Visualization and Computer Graphics* (2018), 1–1. doi:10.1109/TVCG.2018.2855742. 10

[WTS*17] WARE C., TURTON T. L., SAMSEL F., BUJACK R., ROGERS D. H.: Evaluating the Perceptual Uniformity of Color Sequences for Feature Discrimination. In *EuroVis Workshop on Reproducibility, Verification, and Validation in Visualization (EuroRV3)* (2017), Lawonn K., Smit N., Cunningham D., (Eds.), The Eurographics Association. doi:10.2312/eurorv3.20171107. 2, 3, 9, 10

[XH12] XU G., HUANG J. Z.: Asymptotic optimality and efficient computation of the leave-subject-out cross-validation. *The Annals of Statistics 40*, 6 (Dec. 2012), 3003–3030. doi:10.1214/12-AOS1063. 7

[YLL15] YOO M.-J., LEE I.-K., LEE S.: Color Sequence Preserving Decolorization. *Computer Graphics Forum 34*, 2 (2015), 373–383. doi:10.1111/cgf.12567. 5

[ZDM*15] ZOLOTOV A., DEKEL A., MANDELKER N., TWEED D., INOUE S., DEGRAF C., CEVERINO D., PRIMACK J. R., BARRO G., FABER S. M.: Compaction and quenching of high-z galaxies in cosmological simulations: Blue and red nuggets. *Monthly Notices of the Royal Astronomical Society 450*, 3 (July 2015), 2327–2353. doi:10.1093/mnras/stv740. 10

[ZH16] ZHOU L., HANSEN C. D.: A Survey of Colormaps in Visualization. *IEEE Transactions on Visualization and Computer Graphics 22*, 8 (Aug. 2016), 2051–2069. doi:10.1109/TVCG.2015.2489649. 5